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ON CONTACT OF THE CRACK EDGES*

V. B. PEN'KOV and L. A. TOLOKONNIKOV

There is considered an infinite elastic plane with a symmetric crack that can be mapped into a circle by a rational function. It is assumed that opposite edges of the crack come into contact because of loading at infinity. It is also assumed that the vertical displacement component is known on the crack contour in the contact range; there is no tangential stress because of symmetry, and there are no stresses on the free part of the boundary. By means of a suitable selection of two piece-wise-meromorphic functions, the boundary conditions result in a Riemann boundary value problem for the vector functions holomorphic on a plane slit along pairs of circular arcs, and having finite order at infinity. A solution is given in quadratures for the problem formulated. Conditions are written down which define the unknown coefficients of the general solution. A specific example is considered when the crack has the profile of a prolate ellipse.

1. Formulation of the problem. Let us consider a crack with two axes of symmetry, subjected to a load symmetric relative to the same axes.

Let us superpose coordinate axes x, y of a plane z (Fig.1) on the axes of symmetry of the crack, and we assume that the crack edges are in mutual contact on the sections A_1B_1, A_2B_2. Let L_0 denote the crack contour, and L the open contours A_1B_1 and A_2B_2. After deformation, part of the contour $L_0 \setminus L$ is load free, and the component $v = -y$ of the displacement $U = u + iv$ is defined on L. There are no tangential stresses on L because of symmetry. Upon the contact zone approaching the points of crack edge juncture from within, the normal stresses on the contour decrease to zero in a continuous manner.

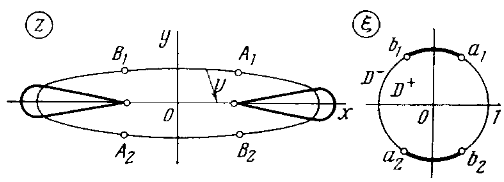

Fig.1

By virtue of the double symmetry for which $\omega(-\xi) = -\omega(\xi)$ and $\omega(\bar{\xi}) = \overline{\omega(\xi)}$, the rational function $z = \omega(\xi)$, which maps the exterior of the contour L_0 conformally on the interior D^+ of the unit circle l_0 in the plane $\xi = |\xi| \exp(i\theta)$, has odd powers and the real coefficients

$$z = \omega(\xi) = A\left(\xi^{-1} + \sum_{j=0}^{k} \xi^{2j+1} a_{2j+1}\right)$$

Let us use the notation $n = 2k + 1$. The mapping mentioned carries a point of the contour L_0 over into the point l_0, in particular, sets up a correspondence between A_1 and $a_1 = a$, B_1 and $b_1 = -\bar{a}$, A_2 and $a_2 = -a$, and B_2 and $b_2 = \bar{a}$.

It is known /1/ that the radial σ_ρ, circular σ_θ, and tangential $\sigma_{\rho\theta}$ stresses in the plane ξ, as well as the displacements u, v in the plane z, are expressed in terms of the complex potentials $\varphi(\xi), \psi(\xi)$, by the dependences ($\varkappa, \mu$ are elastic constants)

$$\sigma_\rho + \sigma_\theta = 2[\Phi(\xi) + \overline{\Phi(\xi)}], \quad \sigma_\rho + \sigma_{\rho\theta} = \Phi(\xi) + \overline{\Phi(\xi)} - \bar{\xi}[\omega(\xi)\overline{\Phi'(\xi)} + \overline{\omega'(\xi)}\Psi(\xi)] \times [\xi\omega'(\xi)]^{-1} \tag{1.1}$$

$$2\mu(u + iv) = \varkappa\varphi(\xi) - \omega(\xi)\overline{\Phi(\xi)} - \overline{\psi(\xi)}, \quad \Phi(\xi)\omega'(\xi) = \varphi'(\xi), \quad \Psi(\xi)\omega'(\xi) = \psi'(\xi)$$

The boundary conditions mentioned for the use of (1.1) are represented in the form

$$F_1(\sigma) \equiv \sigma c Z - \bar{\sigma}\bar{c}\bar{Z} = 0, \quad \sigma \in l_0 \tag{1.2}$$

$$2|\omega'|^2(\Phi + \bar{\Phi}) - \sigma c Z - \bar{\sigma}\bar{c}\bar{Z} = 0, \quad \sigma \in l_0 \setminus l, \quad \Phi(\varkappa c - \bar{c}) + \bar{\Phi}(\varkappa\bar{c} - c) + \sigma Z + \bar{\sigma}\bar{Z} = 4\mu v', \quad \sigma \in l \tag{1.3}$$

*Prikl.Matem.Mekhan,44,No.4,752-759,1980 where the notation $c = \sigma\omega'(\sigma)$, $Z = \overline{\omega(\sigma)}\,\Phi'(\sigma) + \omega'(\sigma)\Psi(\sigma)$, $v' = dv/d\theta = {}^1/_2(c + \bar c)$ has been introduced for brevity and the arguments have been omitted. The conditions (1.3) can be replaced by equivalent conditions by combining them with (1.2)

$$F_2(\sigma) \equiv \bar c(\Phi + \bar\Phi) - \sigma Z = 0, \sigma \in l_0 \setminus l, \quad F_3(\sigma) \equiv \Phi(\varkappa c - \bar c) + \bar\Phi(\varkappa c - c) + \sigma(1 + c/\bar c)Z = 4\mu v', \sigma \in l \tag{1.4}$$

Let us introduce the piecewise-holomorphic vector $\Omega(\xi) = \{\Omega_1, \Omega_2\}$ related to the potentials $\Phi(\xi), \Psi(\xi)$ for $|\xi| < 1$ and their propagations by symmetry $\bar\Phi(\xi^{-1}), \Psi(\xi^{-1})$ through the unit circle l_0 by the formulas

$$\Omega_1(\xi) = \begin{cases} \xi^{-1}\bar\omega'(\xi^{-1})\bar\Phi(\xi^{-1}), & |\xi| > 1 \\ -\xi^{-1}\bar\omega'(\xi^{-1})\Phi(\xi) + \xi\bar\omega(\xi^{-1})\Phi'(\xi) + \xi\omega'(\xi)\Psi(\xi), & |\xi| < 1 \end{cases} \tag{1.5}$$

$$\Omega_2(\xi) = \begin{cases} -\omega'(\xi)\bar\Phi(\xi^{-1}) + \xi^{-2}\omega(\xi)\bar\Phi'(\xi^{-1}) + \xi^{-2}\bar\omega'(\xi^{-1})\bar\Psi(\xi^{-1}), & |\xi| > 1 \\ \omega'(\xi)\Phi(\xi), & |\xi| < 1 \end{cases}$$

Let us consider $F_1(\sigma)$, $F_2(\sigma)$, $F_3(\sigma)$ as values on the contour l_0 for the functions $F_1(\xi)$, $F_2(\xi)$, $F_3(\xi)$. Let us manipulate these latter by using (1.5), and let us pass to the limit as $\xi \to \sigma$. We obtain boundary conditions corresponding to (1.2) and (1.4)

$$c(\Omega_1^+ - \Omega_1^-) + \bar\omega'(\Omega_2^+ - \Omega_2^-) = 0, \quad \sigma \in l_0 \tag{1.6}$$

$$\Omega_1^+ - \Omega_1^- = 0, \quad \sigma \in l_0 \setminus l \tag{1.7}$$

$$(c + \bar c)\Omega_1^+ + (\varkappa\bar c - c)\Omega_1^- + (\varkappa + 1)\bar\omega'\Omega_2^+ = 4\mu v'\bar c, \quad \sigma \in l \tag{1.8}$$

where the plus and minus superscripts denote the limits of the functions as they approach the contour l_0 from D^+ and D^-, respectively.

The relationship (1.6) on the contour $l_0 \setminus l$ has the form

$$\Omega_2^+ - \Omega_2^- = 0, \sigma \in l_0 \setminus l \tag{1.9}$$

and it can be converted on the contour l to the form

$$(\bar c - \varkappa c)\Omega_1^+ + \varkappa(c + \bar c)\Omega_1^- + (x + 1)\bar w'\Omega_2^- = 4\mu v'\bar c, \quad \sigma \in l \tag{1.10}$$

The conditions (1.7)—(1.10) pose a Riemann boundary value problem for the vector $\Omega(\xi)$, which is meromorphic on the circle l_0 slit along the arc l on the plane ξ. After having solved the system (1.8) and (1.10) for $\Omega^+(\sigma)$, this problem can be written in matrix form thus:

$$\Omega^+(\sigma) = G(\sigma)\Omega^-(\sigma) + g(\sigma), \quad \sigma \in l \tag{1.11}$$

$$G(\sigma) = \frac{1}{\varkappa c - \bar c}\begin{Vmatrix} \varkappa(c + \bar c) & (\varkappa + 1)\bar\omega' \\ -(\varkappa + 1)\omega' & -(c + c') \end{Vmatrix}, \quad g(\sigma) = 2\mu\frac{c + \bar c}{\varkappa c - \bar c}\begin{Vmatrix} \bar c \\ -\omega' \end{Vmatrix}$$

The poles of the vector $\Omega(\xi)$ are determined from (1.5): $\Omega_1(\xi)$, $\Omega_2(\xi)$ have poles of the order $n, 1$ and $2, n - 1$, respectively, at zero and infinity. For the piecewise-meromorphic vector $\Omega(\xi)$, the problem (1.11) is equivalent to the problem for a piecewise-holomorphic vector $N(\xi)$, for which the order of the poles at infinity is $n + 1$

$$N^+(\sigma) = G_0(\sigma)N^-(\sigma) + g_0(\sigma), \sigma \in l \tag{1.12}$$

$$N(\xi) = \Xi^{-1}(\xi)\Omega(\xi), \quad G_0(\sigma) = \Xi^{-1}(\sigma)C(\sigma)\Xi(\sigma), \quad g_0(\sigma) = \Xi^{-1}(\sigma)g(\sigma)$$

where $\Xi(\xi)$ is a diagonal matrix with the principal elements ξ^{-n} and ξ^{-2}.

The matrices $G_0(\sigma)$ and $G(\sigma)$ are nonsingular on l. In fact, their determinant

$$\Delta(\sigma) = \det G(\sigma) = \det G_0(\sigma) = (\varkappa\bar c - c)/(\varkappa c - \bar c)$$

vanishes nowhere on l: the modulus of $\Delta(\sigma)$ on l equals one.

2. Solution of the Riemann problem on open contours.

Let us consider the homogeneous problem corresponding to (1.12)

$$N^+(\sigma) = G_0(\sigma)N^-(\sigma), \sigma \in l \tag{2.1}$$

Let us represent the matrix $G_0(\sigma)$ with rational coefficients in the form

$$G_0(\sigma) = R(\sigma)H(\sigma)R^{-1}(\sigma), \quad H(\sigma) = \begin{Vmatrix} 1 & 0 \\ 0 & \Delta(\sigma) \end{Vmatrix}, \quad R(\sigma) = \begin{Vmatrix} -1 & \sigma^n\bar c \\ 1 & \sigma^n c \end{Vmatrix} \tag{2.2}$$

Here $H(\sigma)$ is a diagonal matrix comprised of characteristic functions of the matrix $G_0(\sigma)$, and $R(\sigma)$ is a polynomial matrix nonsingular on l.

For the vector $S(\xi)$ determined by the expression

$$S(\xi) = \det R(\xi)\,R^{-1}(\xi)\,N(\xi) \tag{2.3}$$

we have the boundary value problem

$$S^+(\sigma) = H(\sigma)\, S^-(\sigma),\ \sigma \in l \tag{2.4}$$

Equation (2.4) is a set of two independent Riemann problems for the piecewise-holomorphic functions $S_1(\xi), S_2(\xi)$ on the open contours l

$$S_1^+(\sigma) = S_1^-(\sigma),\ \sigma \in l \tag{2.5}$$

$$S_2^+(\sigma) = \Delta(\sigma)\, S_2^-(\sigma),\ \sigma \in l \tag{2.6}$$

Methods to solve the last problems are known /2/.

We seek canonical solutions of the problems (2.5), (2.6) in the class of functions bounded on the ends of the contours l (let us recall that the stress σ_ρ is zero at these points). The function $S_1(\xi)$ is continuous on l_0 and has one as canonical solution. The index of the problem (2.5) is zero. The problem for $S_2(\xi)$ on the contour l_0 has a discontinuous coefficient $K(\sigma)$ equal to $\Delta(\sigma)$ on l and 1 on $l_0 \setminus l$.

Let us introduce the functions $S_3^+(\xi), S_3^-(\xi)$ by the formulas

$$S_2^\pm(\xi) = \Pi^\pm(\xi)\, S_3^\pm(\xi),\ \Pi^+(\xi) = [(\xi^2 - a^2)(\xi^2 - \bar{a}^2)]^{-\vartheta},\ \Pi^-(\xi) = [(1 - a^2/\xi^2)(1 - \bar{a}^2/\xi^2)]^{-\vartheta} \tag{2.7}$$

The function $\Pi^+(\xi)$ is holomorphic on the plane ξ slit along arbitrary lines connecting the points $\pm a, \pm\bar{a}$ and ∞. The function $\Pi^-(\xi)$ is holomorphic on the plane ξ slit along lines connecting 0 and $\pm a, \pm\bar{a}$. The exponent ϑ characterizes the jump in the coefficient of the problem under consideration during the passage through the ends of l : because of biaxial symmetry they are identical and equal to $\exp(2\pi i \vartheta)$.

The problem

$$S_3^+(\sigma) = K_0(\sigma)\, S_3^-(\sigma),\ \sigma \in l_0 \tag{2.8}$$

has the continuous coefficient $K(\sigma)\Pi^-(\sigma)/\Pi^+(\sigma)$. Let the index of the problem (2.8) be denoted by λ. By definition /2/, the index of the problem (2.8) depends on the class of the solution of $S_3(\xi)$ and on the length of the contact zone l. By virtue of symmetry, it is even. For solutions bounded on the ends of l and a small length a_1b_1, it equals (-2), ($-2m$ if the number of pairs of contact zones is m). Magnification of the length a_1, b_1 can deflect the value of the index to either side, however, we shall consider small zones so that $\lambda \leqslant 0$.

The canonical solution of the problem (2.8) has the form /2/

$$y^+(\xi) = \exp\Gamma^+(\xi),\ y^-(\xi) = \xi^{-\lambda}\exp\Gamma^-(\xi),\quad \Gamma(\xi) = \frac{1}{2\pi i}\int_{l_0}\frac{\ln K_0(\sigma)\,d\sigma}{\sigma - \xi} \tag{2.9}$$

We obtain the canonical solution of the problem (2.6) on the basis of (2.7) and (2.9):

$$x(\xi) = \Pi(\xi)\, y(\xi) \tag{2.10}$$

Therefore, the canonical solution of the problem (2.4) has the form

$$X(\xi) = \begin{Vmatrix} 1 & 0 \\ 0 & x(\xi) \end{Vmatrix} \tag{2.11}$$

We see that the matrix $R(\xi)X(\xi)$ is the funamental solution of the problem (2.1). In fact

$$H(\sigma) = X^+(\sigma)[X^-(\sigma)]^{-1}$$

Then

$$G_0(RX^-) = RHR^{-1}RX^- = RX^+[X^-]^{-1}X^- = RX^+$$

Moreover

$$\det(RX) = \det R(\xi)\det X(\xi) \not\equiv 0$$

since the matrix $R(\xi)$ is nonsingular on l, and therefore its determinant is not identically zero, Q.E.D.

The determinant of the matrix $R(\xi)$ can be represented in the form

$$\delta(\xi) = \det R(\xi) = \xi^n[\bar{c}(\xi^{-1}) - c(\xi)] = \delta_0\prod_{j=1}^{2n}(\xi - \xi_j)$$

The process of constructing the normal system of solutions from the fundamental consists of a finite number of operations on the successive elimination of the zeroes of $\delta(\xi)$ from a finite part of the plane /3/. For the matrix $R(\xi)X(\xi)$ the set of these operations is equivalent to multiplying it on the right by a matrix $Q(\xi)$ whose structure has the form

$$Q(\xi) = \begin{Vmatrix} 1 & q(\xi)/\delta(\xi) \\ 0 & 1/\delta(\xi) \end{Vmatrix}$$

where $q(\xi)$ is a polynomial of order not higher than $2n - 1$. The coefficients of the polynomial are determined by the process mentioned above.

The matrix of the normal system of solutions has the form

$$T(\xi) = RXQ = \begin{Vmatrix} -1 & [-q(\xi) + \xi^n \bar{c}(\xi^{-1}) x(\xi)] / \delta(\xi) \\ 1 & [q(\xi) - \xi^n c(\xi) x(\xi)] / \delta(\xi) \end{Vmatrix}$$

The order /3/ of the first column of the matrix $T(\xi)$ equals zero, while the order of the second equals $-\lambda$ for $\lambda \leqslant 0$. Let us consider the limit

$$\lim_{\xi \to \infty} \det \left\{ T(\xi) \begin{Vmatrix} 1 & 0 \\ 0 & \xi^\lambda \end{Vmatrix} \right\} = \det \begin{Vmatrix} -1 & 0 \\ 1 & B \end{Vmatrix} = -B$$

where B is the coefficient of the highest term $\xi^{-\lambda}$ in the expansion $T_{22}(\xi)$ in powers of ξ in the neighborhood of the infinitely remote point. Since $B \neq 0$, the normal system of solutions of $T(\xi)$ is then canonical /3/, where the particular indexes are 0 and λ.

The general solution of the inhomogeneous Riemann problem (1.12) has the form

$$N(\xi) = T(\xi) \left\{ \frac{1}{2\pi i} \int_l \frac{[T^+(\sigma)]^{-1} g_0(\sigma)}{\sigma - \xi} d\sigma + P(\xi) \right\} \tag{2.12}$$

where $P(\xi) = \{P_1, P_2\}$; $P_1(\xi), P_2(\xi)$ are polynomials of orders $n+1$ and $n+1+\lambda$ with undetermined coefficients.

The general solution of the problem (1.11) is represented in the form

$$\Omega(\xi) = Y(\xi) \left\{ \frac{1}{2\pi i} \int_l \frac{[Y^+(\sigma)]^{-1} g(\sigma)}{\sigma - \xi} d\sigma + P(\sigma) \right\}, \quad Y(\xi) = \Xi(\xi) T(\xi) \tag{2.13}$$

Among all the sets of undertermined coefficients of the vector $P(\xi)$, there exists that which satisfies the conditions of the elasticity theory problem posed, namely:

a) The condition (the corollary (1.5)) should be satisfied

$$\xi \, \Omega_1(\xi) \equiv \overline{\Omega}_2(\xi^{-1}) \tag{2.14}$$

b) The potentials $\Phi(\xi), \Psi(\xi)$ should have the following form in the neighborhood of $\xi = 0$ /1/:

$$\Phi(\xi) = \Gamma + \Phi_0(\xi), \quad \Psi(\xi) = \Gamma' + \Psi_0(\xi) \tag{2.15}$$

where Γ, Γ' are quantities characterizing the load at infinity; $\Phi_0(\xi), \Psi_0(\xi)$ are holomorphic functions in the neighborhood of $\xi = 0$.

c) Since its derivative, rather than the displacement itself, is given in the initial boundary conditions, the displacement is then defined by a solution to the accuracy of an arbitrary constant, and the juncture condition for the crack edges should be satisfied

$$U[\omega(a)] - U[\omega(\bar{a})] = \omega(\bar{a}) - \omega(a) \tag{2.16}$$

Upon using the conditions (2.14) and (2.15), the elements of the matrix $T(\xi)$ should be expanded in a Taylor series in the neighborhood of $\xi = 0$. In addition, the integrals being understood in (2.13), are subject to expansion in a power series in ξ. Upon using conditions (2.15), it should be taken into account that the potentials $\Phi(\xi)$ and $\Psi(\xi)$ are expressed in terms of $\Omega(\xi)$ in the form

$$\Phi(\xi)\,\omega'(\xi) = \Omega_2(\xi), \ |\xi| < 1, \ \xi\omega'(\xi)\Psi(\xi) = \Omega_1(\xi) + \xi^{-1}\bar{\omega}'(\xi^{-1})\Phi(\xi) - \xi\bar{\omega}(\xi^{-1})\Phi'(\xi), \ |\xi| < 1$$

The relationships (2.16) can be rewritten in the form

$$2\mu[\omega(\bar{a}) - \omega(a)] = i \int_{\bar{a}}^{a} [c(\varkappa\Phi - \overline{\Phi}) + \bar{\sigma}\bar{Z}] d\sigma \tag{2.17}$$

Formula (2.17) is obtained by differentiation of the last equation of (1.1) and using the notation taken in (1.3)

The calculational labour can be reduced significantly if the results in /4/ are used: For problems possessing biaxial symmetry, expansions of the complex potentials $\Phi(\xi), \Psi(\xi)$ in powers of ξ have only even powers and real coefficients. The functions: $x(\xi)$; elements $T_{12}(\xi), T_{22}(\xi)$ of the second column of the matrix $T(\xi)$, the integrals $I^1(\xi), I^2(\xi)$ being understood in (2.13), indeed possess the properties of such symmetry. Confirmation of this assertion consists of confirming compliance of the equalities $F(-\xi) = F(\xi), F(\xi) = \overline{F(\bar{\xi})}$ for each of these functions. Hence, expansions with real coefficients are valid

$$x(\xi) = x_0 + x_2\xi^2 + x_4\xi^4 + \ldots, \quad I^1(\xi) = I_0^1 + I_2^1\xi^2 + \ldots, \quad I^2(\xi) = I_0^2 + I_2^2\xi^2 + \ldots \tag{2.18}$$

$$Y(\xi) = \begin{Vmatrix} -\xi^{-1} & \xi^{-1}(t_0^1 + t_2^1\xi^2 + \ldots) \\ \xi^{-2} & \xi^{-2}(t_0^2 + t_1^2\xi^2 + \ldots) \end{Vmatrix}$$

Now, a deduction can be made from (2.13) that odd coefficients of the polynomials $P_1(\xi)$, $P(\xi)$ are zero, while the even are real.

It should be noted that the load at infinity should be matched to the width of the contact zone; for a given load the normal stress at the point a of the contour l_0 should be zero (the line is taken from inside l).

Remark 1°. The computations remain in force if the number of pairs of contact zones is m. Here only the function $\Pi(\xi)$ that has the following form in this case

$$\Pi^{\pm}(\xi) = \prod_{j=1}^{m} \Pi_j^{\pm}(\xi)$$

varies, and $\Pi_j^{\pm}(\xi)$ are understood to be the expressions in the right sides of the formulas considered in (2.7). It should be taken into account that a is now understood to be a_j, the beginning of the zone j, and ϑ is understood to be ϑ_j, a constant characterizing the jump in the coefficient $K_0(\sigma)$ at the point a_j.

2°. Upon constructing the canonical solution of the problem (2.1), the condition $\lambda \leqslant 0$, taken from physical considerations, is imposed on the index of the particular problem (2.6). In general, any value of λ, particularly $\lambda > 0$ is allowable. In the latter case, $T(\xi)$ will not be a canonical solution and definite procedures /3/ will be required to construct this latter. The set of procedures on the construction of a canonical solution can be replaced by a matrix factor acting on the normal solution of $T(\xi)$ from the right.

The subsequent reasoning remains valid.

3. Example. Crack of elliptical profile.

Let the crack have a width $2d$ and a span $2D$. The crack contour is mapped into the unit circle l_0 by the functions

$$z = \omega(\xi) = A(\xi^{-1} + m\xi), \quad A = 1/2(D-d), \quad m = (D+d)/(D-d) > 1$$

Equation (1.1) has the coefficient $G(\sigma)$ and free term $g(\sigma)$ in the form

$$G(\sigma) = \frac{1}{(\varkappa+m)(\sigma\varepsilon^2 - \sigma^{-1})} \begin{Vmatrix} \varkappa(m-1)(\sigma+\sigma^{-1}) & (\varkappa+1)(m-\sigma^2) \\ (\varkappa+1)(\sigma^{-2}-m) & (1-m)(\sigma+\sigma^{-1}) \end{Vmatrix}$$

$$g(\sigma) = 2\mu A \frac{m-1}{\varkappa+m} \frac{\sigma^2+1}{\sigma^2(\sigma^2\varepsilon^2-1)} \begin{Vmatrix} \sigma(\sigma^2-m) \\ m\sigma^2-1 \end{Vmatrix}, \quad \varepsilon^2 = \frac{1+\varkappa m}{\varkappa+m}$$

$$\det G(\sigma) = -\varepsilon^2(\sigma^2-\varepsilon^2)/(\sigma^2 - 1/\varepsilon^2)$$

The matrix $R(\xi)$ and the normalizing factor $Q(\xi)$ have the form

$$R(\xi) = \begin{Vmatrix} -1 & A(m-\xi^2) \\ 1 & A(1-w\xi^2) \end{Vmatrix}, \quad Q(\xi) = \begin{Vmatrix} 1 & Ae/\delta(\xi) \\ 0 & 1/\delta(\xi) \end{Vmatrix}$$

$$\delta(\xi) = A(m+1)(\xi^2-1), \quad e = (m-1)x(1)$$

where $x(1)$ is evaluated by means of (2.10). Since $n=1$, then the matrices $\Xi(\xi)$, $P(\xi)$ have the form

$$\Xi(\xi) = \begin{Vmatrix} \xi^{-1} & 0 \\ 0 & \xi^{-2} \end{Vmatrix}, \quad P(\xi) = \begin{Vmatrix} P_0^1 + P_1^1\xi + P_2^1\xi^2 \\ P_0^2 \end{Vmatrix}$$

Because of the above $P_1^1 = 0$; P_0^1, P_2^1, P_0^2 are real.

Let us determine the unknown coefficients by using the expansions (2.18). The condition (2.14) adds nothing new; it just confirms the real nature of the coefficients. Representation of the potentials $\Phi(\xi)$, $\Psi(\xi)$ in the form (2.15) yields two coefficients:

$$P_0^1 = -I_0^1 + A\{\Gamma'[(m-1)x(1) + x(0)] + \Gamma(m^2-1)x(1)\} \times [x(0)(m+1)]^{-1}, \quad P_0^2 = -I_0^2 + A\{\Gamma' + (m+1)\Gamma\}[x(0)]^{-1}$$

As a result of integrating, the relationship (2.17) becomes

$$2\mu[\omega(\bar{a}) - \omega(a)] = P_2^1 w_1 + w_2$$

where ω_1, ω_2 are some constants. This latter expression determines P_2^1 uniquely. In order to match the load at infinity with the width of the contact zone, we consider Γ and Γ' functions of the parameter γ. Then the vector $\Omega(\xi)$, therefore, $\Phi(\xi)$ and $\Psi(\xi)$ will also be functions of this parameter. The expression of the normal stress $\sigma_\rho(\xi, \gamma)$ in terms of the latter is given by (1.1) and (1.5). Solving the equation

$$\sigma_\rho(a, \gamma) = 0$$

we obtain a value of the parameter γ matching the value under consideration.

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